Section 3.5: Subspaces Associated to a Matrix

Let $A \in M_{m,n}(\mathbb{R})$.

- The **null space** of A is defined as

$$\mathrm{NS}(A) = \{\vec{\mathbf{x}} \in \mathbb{R}^n \,|\, A\vec{\mathbf{x}} = \vec{0}\}.$$

- The **column space** of A is the span of the columns of A, considered as vectors in $\mathbb{R}^m$. The column space of A is denoted by $\mathrm{CS}(A)$.
- The **row space** of A is the span of the rows of A, considered as vectors in $\mathbb{R}^n$. The row space of A is denoted by $\mathrm{RS}(A)$.

Bases for Subspaces Associated to a Matrix

Let $A \in M_{m,n}(\mathbb{R})$.

- (Theorem 3.48) To find a basis for $\mathrm{NS}(A)$, first solve $A\vec{\mathbf{x}} = \vec{0}$ and parameterize the solution set with parameters $t_1, t_2, \ldots, t_k$. Write this solution as

$$\vec{\mathbf{x}} = \vec{\mathbf{x}}_1 t_1 + \vec{\mathbf{x}}_2 t_2 + \cdots + \vec{\mathbf{x}}_k t_k.$$

  The set $\{\vec{\mathbf{x}}_1, \vec{\mathbf{x}}_2, \ldots, \vec{\mathbf{x}}_k\}$ is a basis of $\mathrm{NS}(A)$.
- The columns of A corresponding to leading columns in $\mathrm{rref}\,(A)$ can be combined to form a basis of $\mathrm{CS}(A)$. (See Theorem 3.47)
- (Theorem 3.49) The nonzero rows of $\mathrm{rref}\,(A)$ form a basis for $\mathrm{RS}(A)$.
- It is important here to use the columns of the original matrix A for the basis of $\mathrm{CS}(A)$ and the rows of $\mathrm{rref}\,(A)$ for the basis of $\mathrm{RS}(A)$.